As can be seen, the improvement arising from the use of the nonasymptotic boundary conditions (7) is significant and the actual and estimated errors are in reasonable agreement. The procedure we have used in this Note may be applied to a variety of boundary-value problems with asymptotic boundary conditions, the principal requirements being that some foreknowledge of the asymptotic behavior of the solutions is needed and that the outer differential equations can be reduced to homogeneous form.

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Turbulent Boundary-Layer Thicknesses on Yawed Cones

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Nomenclature

b = blowing parameter = $[(\rho v)_w/(\rho u)_e]2/C_{f_0}$
C_f = skin friction coefficient
L = model length
M = Mach number
MW = molecular weight
Re = Reynolds number based on model length
T = temperature
T^* = reference temperature
u = velocity parallel to model wall
v = velocity normal to model wall
α = angle of attack
δ = boundary-layer thickness
$\bar{\delta}$ = $[\delta(\alpha)/\delta(0)]/[\delta_c(\alpha)/\delta_c(0)]$
θ = momentum thickness
θ_c = cone half angle
ρ = density

Subscripts

∞ = freestream conditions
c = critical or calculated
e = boundary-layer edge conditions
0 = without mass addition
r = recovery conditions
w = wall conditions

Introduction

INFORMATION on the growth and thickness of boundary layers is required in order to estimate viscous interaction effects, plasma attenuation, and wake characteristics. For unyawed cones, Ref. 1 is widely used for making engineering calculations of turbulent boundary layer properties both with and without mass addition (blowing). In the present Note data are presented which indicate the effect of α on δ along the windward and leeward rays of a yawed cone. Test conditions include M_∞'s ranging from 1.8 to 10.2, cases with and without heat transfer and with and without blowing. It is shown that the effect of crossflow on δ for all of these data can be satisfactorily represented by a single curve.

Experimental Data

Test conditions for the experimental data considered are summarized in Table 1. The new data without blowing reported herein were obtained in Tunnels *B* and *C* of the Arnold Engineering Development Center. Nominal reservoir conditions were 220 psia and 850°R at $M_\infty = 6.05$ and 1725 psia and 1900°R at $M_\infty = 10.2$. The test model was a 36-in. long, 9° half-angle cone equipped with a boundary-layer trip having a roughness height of 0.080 in. located near its nose. In one series of experiments the nose was sharp and wall temperature was near adiabatic, whereas in another series of experiments the nose radius was 0.15 in. and there was heat transfer to the wall.

The blowing data were taken on a 10-in. long, 9° half-angle cone equipped with a porous skin through which air could be ejected. The model had a nose radius of 0.053 in. and was tested without a boundary layer trip. A $M_\infty = 8$ airstream was generated by 1000 psia and 1340°R reservoir conditions. Without blowing the model boundary-layer was laminar; however, for the blowing tests the boundary layer became turbulent near the nose. Additional details on the experimental setup are presented in Ref. 2.

The δ's reported herein were measured at the model base. For the AEDC test the δ's represent averages of measurements taken from shadowgraph or color schlieren photographs from several runs. These results are summarized in Table 2. The δ's from the Ref. 2 test were determined from shadowgraph pictures whereas the δ's reported in Ref. 3 were determined from pitot pressure surveys. All of the δ's obtained from the blowing test are included in Table 3.

Results and Discussion

The windward data from Tables 2 and 3 and Ref. 3 are plotted in Fig. 1. The change in δ with α exhibited in Fig. 1 can be separated into two effects, 1) changes in δ caused by changes in local flow conditions as the cone is yawed, and 2) thinning of the boundary layer due to crossflow. In order to isolate these effects, changes in δ caused by changes in local conditions were calculated. The method of Ref. 1 was used with the following modifications which were obtained from

Table 1 Summary of test conditions

Symbol	Source	M_∞	T_w/T_r	θ_c (deg)	L (in.)	Re_∞ (10^6)	$\frac{(\rho v)_w}{(\rho u)_\infty}$
◇	Present Data	6.05	1	9	36	11.8	0
⊙		10.2	1	9	36	6.3	0
●		10.2	0.37 to 0.52	9	36	6.3	0
⊡	Ref. 2	8.0	0.69	9	10	4.0	0.013
■		8.0	0.55	9	10	4.0	0.017
△	Ref. 3	1.8	1	12.5	41.6	25	0
▽		4.25	1	12.5	41.6	51	0

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Table 2 Summary of experimental results without mass addition

				Measurements	
M_∞	Data source	α, deg	No.	Average, in.	Range in δ, in.
10.2	Shadowgraph	0	17	0.353	0.315–0.405
		8	8	0.240	0.202–0.263
		10	8	0.214	0.150–0.248
		15	2	0.176	0.150–0.202
		20	2	0.124	0.098–0.150
6.05		0	16	0.293	0.258–0.349
		5	4	0.204	0.176–0.205
		8	8	0.176	0.170–0.181
		10	4	0.135	0.132–0.147
		15	10	0.111	0.091–0.132
		20	4	0.103	0.097–0.115
		25	9	0.081	0.060–0.091
10.2	Schlieren	0	4	0.376	0.360–0.402
		8	3	0.228	0.227–0.230
		15	3	0.197	0.187–0.190
		27	3	0.083	0.074–0.095

Ref. 4;

$$\frac{C_f}{C_{f_0}} = \frac{(1 - b/b_c)^{2.0}}{(1 + b/b_c)^{0.4}} \qquad (1)$$

where‡

$$b_c = 12/(\{1 + [2(MW)_e/(MW)_w]T_w/T_e\})(T^*/T_e)^2 \qquad (2)$$

Local conditions used to calculate boundary-layer parameters were inviscid tangent cone values except for $-1.0 \geq \alpha/\theta_c$ where freestream conditions were used. No attempt was made to correct for boundary-layer displacement effects although these effects were not negligible for runs with mass addition.

The calculated effect of local conditions was found to be small for the data without blowing ($0.96 \leq \delta_c(\alpha)/\delta_c(0) \leq 1.08$). In the tests with mass addition the blowing was strong enough to increase θ and b by an order of magnitude at $\alpha = 0$. However, since $(\rho v)_w$ was held constant, the calculated influence of blowing on θ and δ decreased with increasing α because b decreased. Consequently, $\delta_c(\alpha)/\delta_c(0)$ was substantially reduced for the tests with mass addition [i.e., $0.46 \leq \delta_c(\alpha)/\delta_c(0) \leq 0.77$]. A parameter $\bar{\delta}$, which includes the effects of crossflow on δ, was formed by dividing $\delta(\alpha)/\delta(0)$ by $\delta_c(\alpha)/\delta_c(0)$. The data of Fig. 1 are re-plotted in terms of $\bar{\delta}$ in Fig. 2. Figure 2 illustrates that apart from its dependence on α/θ_c that $\bar{\delta}$ is essentially independent of a wide range of environmental conditions.

Table 3 Experimental results with mass addition

M_∞	Data source	$(\rho v)_w/(\rho u)_\infty$	α, deg	Measured δ, in.
8.04	Shadowgraph	0.013	−10	1.790
			−5	1.035, 1.120
			0	0.560, 0.644
			5	0.308, 0.308
			10	0.210
			15	0.154, 0.168
		0.017	−10	1.988
			−5	1.342
			0	0.700, 0.770
			5	0.364
			10	0.266
			15	0.168
			20	0.154

‡ The reference temperature ratio term in Eq. (2) was not included in Ref. 4. It was suggested by C. P. Gardiner of McDonnell Douglas to provide better agreement with experimental results.

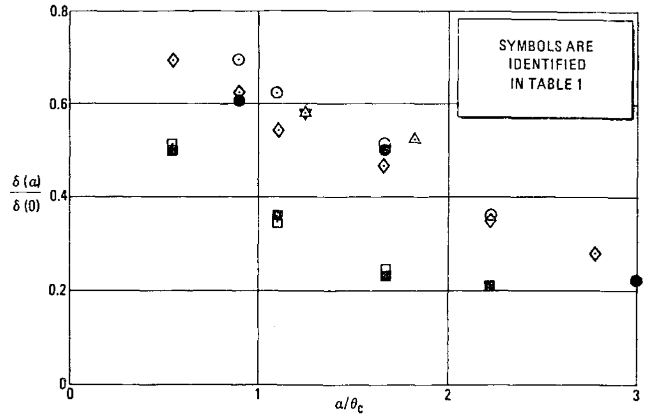

Fig. 1 Variation of turbulent boundary-layer thickness on windward ray of a yawed cone.

Figure 3 contains all of the Fig. 2 data plus some lee-side results from the blowing experiment. The broken line is

$$\ln\bar{\delta} = -2.0 + 2.0 \exp(-0.3\alpha/\theta_c) \qquad (3)$$

which is an adequate fit to the data for engineering purposes.

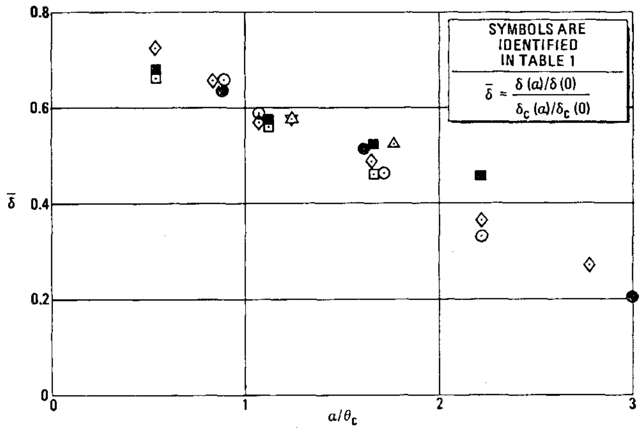

Fig. 2 Effect of crossflow on turbulent boundary-layer thickness on windward ray of a yawed cone.

Much of the data that form the basis for Eq. (3) were obtained for circumstances that were less than ideal for obtaining a correlation of δ's. Specifically, the new no-blowing data were obtained with a boundary-layer trip, and all δ's except those from Ref. 3 were obtained from photographs (instead of detailed probes of the boundary layer).

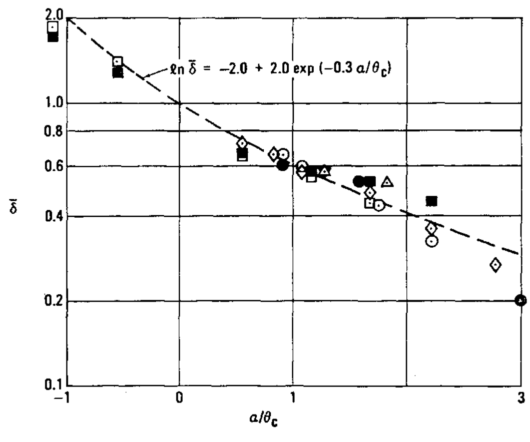

Fig. 3 Effect of crossflow on cone pitch plane turbulent boundary-layer thickness.

In spite of these shortcomings, it is believed that the use of Eq. (3) is preferable to the alternatives of either assuming $\bar{\delta} = 1$ or using Tracy's laminar results.[5]

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Some Observations in the Near Wake of Blunt Bodies

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Nomenclature

c' = velocity fluctuation
D = diameter of body
D_w = wake diameter (width)
f = shedding frequency
H = height of body
P_B = base pressure
P_s = stagnation pressure
R = space correlation
Re_D = Reynolds number (DU_∞/ν)
S = general Strouhal number (fD/U_∞)
S_w = universal wake Strouhal number (fD_w/U_w)
U_w = wake velocity $(U_\infty(1 - p_B/p_s)^{1/2})$
U_∞ = undisturbed ambient velocity
x,y,z = coordinates
Λ = integral scale

Introduction

INTEREST in the flow structure behind blunt, i.e., finite cylinders and cones placed perpendicular into a parallel airstream stems from practical cases like crosswind interference at rocket launching as well as the aerodynamics of structures and wake effects behind buildings. It is obvious that the great number of governing parameters and the fact that this is a transient kind of flow will limit the possibility of generalization of measurements in this kind of flow to some extent and thus its scientific value. For this reason little research has been done on this problem in contrast to the well investigated and documented far wake of an infinite cylinder. From practical considerations, however, it appeared worth while to obtain at least some general characteristics.

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Experimental Investigations

The measurements were performed in the open test section blower tunnel of the Hermann Föttinger-Institut at the Technische Universität Berlin and in the Wright Brothers Wind Tunnel of the Massachusetts Institute of Technology. Truncated circular cylinders with aspect ratios of 4 and 8 as well as cones served as models which were mounted on a flat plate fixed in the x-y plane in the test section with their axis at right angle to the shear free flow, (z direction). Most of the measurements were restricted to the region from 0-5 obstacle diameters downstream and Reynolds numbers less than 10^5 (subcritical range).

Flow Visualization

To study the flow pattern in a general way the flow was rendered visible by smoke. Surface flow patterns were obtained with a thin coating of paraffin lamp-black mixture. In case of the blunt cylinder they exhibited a remarkable straight separation line up to the top for all Reynolds numbers investigated.

Measurements of Wake Characteristics

A more detailed investigation was carried out in the wake of a cylinder with $H/D = 4$ at $Re_D = 4\ 10^4$ (subcritical; nonperiodic wake). Static pressure distributions on the model surface were measured at three different heights around the circumference. By use of a special hot wire probe consisting of five wires the mean flow velocity and its components were measured at various stations in the wake. The turbulence levels in the wake as obtained with a conventional hot wire reached values of up to 80% of the local mean velocity. Since in flows with such extreme conditions hot wire data are no longer correct, these results can only serve for general interpretations.

Two-point correlations of the c'-signal, R_x, R_y, and R_z, were measured and from these the integral scales evaluated. It was found that from $x/D = 2$ on the mean values of the macro scales over the cylinder height $\bar{\Lambda}/x$ = const, being between 0.1 and 0.15 (Fig. 1). In the region very near the cylinder surface, the average turbulent "lump" is stretched in z direction. Further downstream it seems to have undergone a stretching by the mean shear which eventually leads to $\bar{\Lambda}_x > \bar{\Lambda}_y > \bar{\Lambda}_z$. Comparative correlation measurements behind the same cylinder fitted with an end plate at the same Reynolds number reveal that the macro scales in the wake of the infinite cylinder with shedding behavior are much larger than those of the finite cylinder with no regular separation. They are in good agreement with measurements reported by El Baroudi,[1] showing integral scales of the order of 10 D.

Spectral Measurements

To investigate the separation behavior of the flow at the cylinder some spectral measurements of the fluctuation with the probe located at $z/H = 0.5$, $x/D = 1.0$, $y/D = 0.5$ were conducted. For comparative reasons the measurements were made not only with the truncated cylinders but in addition

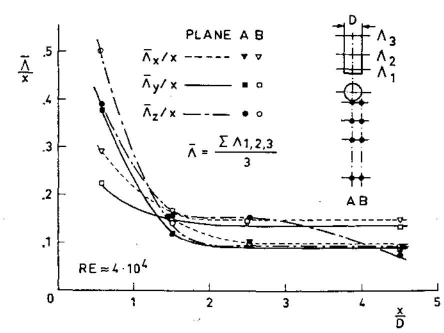

Fig. 2 Integral scales behind finite cylinder.